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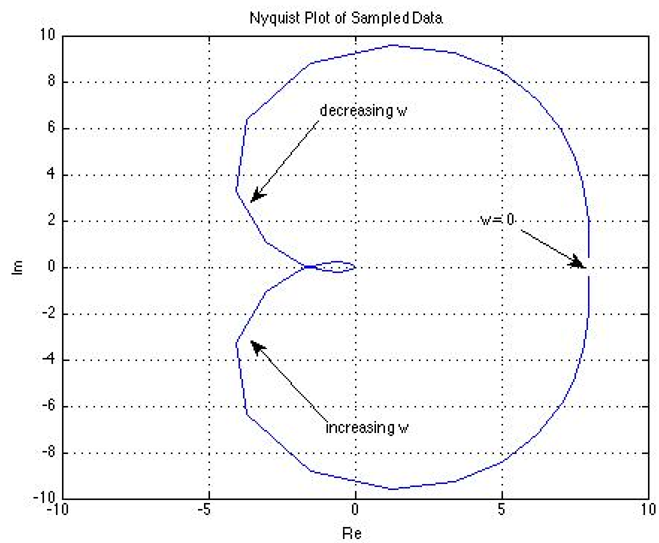
Bonus HW

A ME 455
November 30, 2008

Dr. Wong plans to design a feedback control system. Without knowing the transfer function of the plant, a set of experiments was performed to estimate the frequency response of the open-loop system.

a) Using the data, plot the Nyquist diagram. Is the system stable if Dr. Wong performs unity feedback of the system?

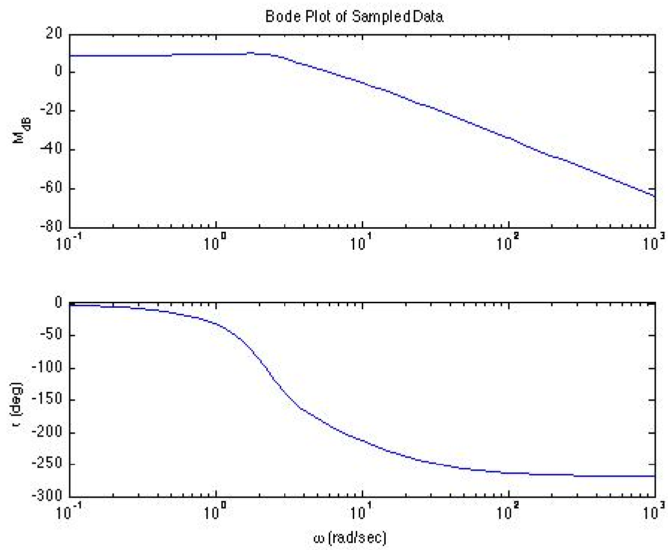
Solution. The Nyquist diagram for the system is



From the chart, it is clear that the open loop system is stable because increasing frequency drives gain to 0. However, the system is *NOT* stable under unity feedback since the Nyquist contour encircles the point $-1 + j0$ twice counterclockwise, which shows that the system fails to satisfy the Nyquist stability criterion.

b) Estimate the Bode plot using the data. What are the gain and phase margins?

Solution. The Bode plot of the sampled data is



The gain margin is the separation in dB of the modulus of $G(jw)$ at -180^o from the modulus at the crossover frequency. A quick look at the plot yields

$$GM \approx -4.1 \text{ dB}.$$

The phase margin is the number of degrees above -180^o the phase is at the crossover frequency. A quick calculation from the graph is

$$PM \approx -9.5^o$$

c) Using the Bode plot, can you estimate the open loop function? Compare your result with the experimental data. [Hint: You can estimate the static gain and system type first, and use the lowest order function to simulate the frequency response. Some trial and error may be required.]

Solution. The system resembles a type-0 system given the behavior on the Bode plot. The open-loop system is also 3rd order, since the phase goes to -270^o. Assume the transfer function has the following form:

$$G(s) = \frac{1}{a_3 s^3 + a_2 s^2 + a_1 s + a_0}.$$

Subbing $s = j\omega$ and solving for the real and imaginary parts yields

$$\begin{aligned} \text{Re}(G) &= \frac{(a_0 - a_2\omega^2)}{(a_0 - a_2\omega^2)^2 + \omega^2(a_1 - a_3\omega^2)^2} \\ \text{Im}(G) &= -\frac{\omega(a_1 - a_3\omega^2)}{(a_0 - a_2\omega^2)^2 + \omega^2(a_1 - a_3\omega^2)^2}. \end{aligned}$$

Interpolate the data given to find where the Nyquist chart crosses the real- and imaginary-axes. This leads to the matrix equation

$$\begin{pmatrix} 1 & 0 & -30.3601 & 0 \\ 0 & 1 & 0 & -30.3601 \\ 1 & 0 & -4.2920 & 0 \\ 0 & 2.0171 & 0 & -8.919 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} -0.6551 \\ 0 \\ 0 \\ 0.1085 \end{pmatrix}$$

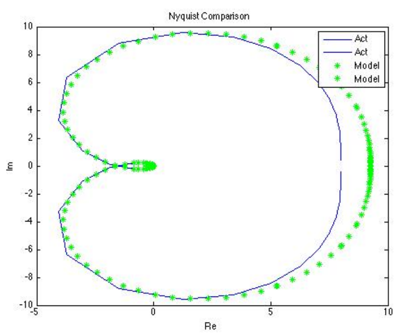
which has the solution

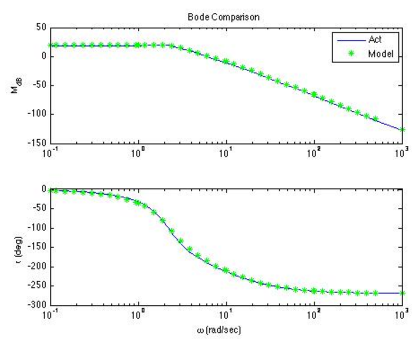
$$\begin{pmatrix} a_3 \\ a_2 \\ a_1 \\ a_0 \end{pmatrix} = \begin{pmatrix} 0.0020 \\ 0.0251 \\ 0.0610 \\ 0.1079 \end{pmatrix}$$

The plant transfer function can then be written as

$$G(s) = \frac{1}{.0020s^3 + .0251s^2 + .0610s + .1079}.$$

As a check, compare this transfer function to the open loop data. The Nyquist and Bode plots are





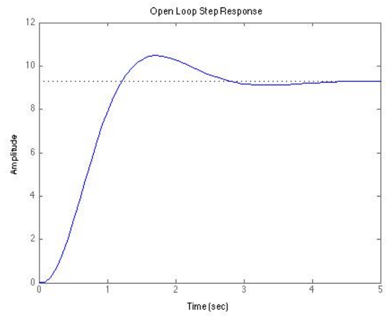
The Bode plot matches almost exactly while the Nyquist plot matches closely for frequencies greater than about 1 rad/sec. The gain and phase margins for the transfer function above are

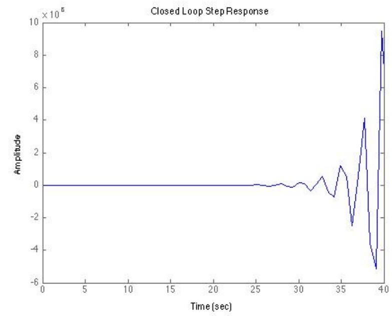
$$\begin{aligned} GM_{\text{dB}} &= -3.622 \text{ dB} \\ PM &= -10.0^o \end{aligned}$$

which are in very close agreement to those values found in part b).

d) Estimate the step response of the open-loop and closed-loop systems.

Solution. Using the **step** command from matlab using the transfer function found in part c). The estimated step responses for the open loop and closed loop system. As a check, compare this transfer function to the open loop data. The Nyquist and Bode plots are

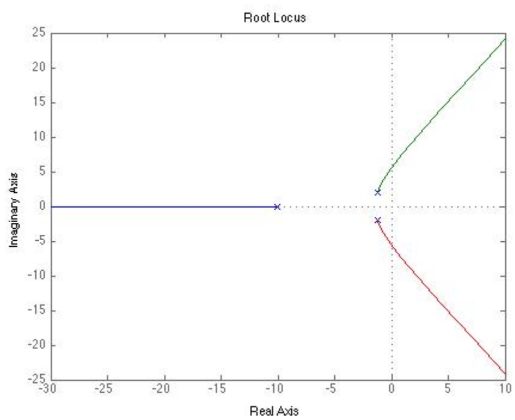




So, unit feedback makes the system unstable. Now I need a feedback controller that will keep the system stable.

e) Draw the root locus for your transfer function.

Solution. The root locus plot for the derived transfer function is



f) Design a controller for your system using the root locus method.

Solution. The crossover gain where the system becomes stable is around $K = .65$. Depending on the requirements for the controller, of which there are none for this assignment, there will be different input responses. Since there are no requirements, I will pick $K = .05$ The system will be stable for this gain and it will minimize the "ringing" of the step response.

g) Estimate the step response of the system with your controller and determine the settling time, rise time, percentage overshoot for the controller you designed.

Solution. The step response with the closed loop gain from part e) is